Interactions of meteoroids with the Earth's atmosphere

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“A space rock a few metres across exploded in Earth’s atmosphere above the city of Chelyabinsk, Russia today at about 03:15 GMT. The numerous injuries and significant damage remind us that what happens in space can affect us all…”

Source: www.esa.int (News, 15 February 2013)
Outline

- Introduction
- A new method implemented for parameters’ identification
- Result testing
- General statistics and consequences
Basic definitions

- **A meteoroid** is a solid object moving in interplanetary space, of a size considerably smaller than an asteroid and larger than an atom.

- **A meteor** is an event produced by a meteoroid entering the atmosphere. On Earth, most meteors are visible in altitude range 70 to 100 km.

- **A fireball** (or bolide) is essentially a bright meteor with larger intensity.

- **A meteorite** is a part of a meteoroid or asteroid that survives its passage through the atmosphere and impacts the ground.
Past terrestrial impacts

The largest verified impact crater on Earth (Vredefort) ~300 km
Past terrestrial impacts

Different types of impact events, examples: formation of a massive single crater (Vredefort, Barringer)
Past terrestrial impacts

Different types of impact events, examples: formation of a massive single crater (Vredefort, Barringer, Lonar Lake)
Past terrestrial impacts

Dispersion of craters and meteorites over a large area (Sikhote-Alin)
Past terrestrial impacts

Tunguska
Tunguska after 100 years...

Area over 2000 km²
Impacts as a hazard
Another reason why we are interested

The three forms of extraterrestrial bodies. Meteorites (right) are remnants of the extraterrestrial bodies (asteroids, comets, and their dusty trails, left) entering Earth’s atmosphere and forming an event called a meteor or fireball (middle). The dark fusion crust on the meteorite is a result of ablation during atmospheric entry.
What can we do in the lab?

Nondestructive physical properties, including measurements of extraterrestrial materials:

- BRF measurements
- Bulk and grain density (i.e. also porosity)
- Magnetic susceptibility
- Internal structure (x-ray microtomography)
- Development of methods for nondestructive thermal and mechanical properties measurements
- Some of the parameters can be determined at wide temperature range (5-1080 K)

Our combined database contains thousands of individual meteorites. Results are published and/or shared within the scientific community.
Example of recent results (1/2)

The FIGIFIGO measuring BRF of selected sample Gibeon (left). The active optics system is located horizontally at the top of the measuring arm, and is looking down to the target through a mirror. FIGIFIGO consists of the following main components: casing, measurement arm, rugged computer, and a sunphotometer on a tripod. The casing contains the main sensor ASD FieldSpec Pro FR optical fiber spectroradiometer (350 – 2500 nm), most of the electronics, and batteries.

Example of measured albedo:
Bruderheim L6 meteorite
Magnetic susceptibility was used as a tool to identify of non-ureilitic meteorites within Almahata Sitta (2008 TC\textsubscript{3}) meteorite collection.

Porosity mapping using XMT.
Classical double-station program

allow us to calculate:
- meteor height
- length along trajectory
- meteor intensity
- spectrum

first attempts:
- Harvard Meteor Project
- Ondrejov Observatory in Czechoslovakia

first success:
- bright fireball photographed on April 7, 1959: four meteorites were found near Příbram in Czechoslovakia in the area predicted from the double-station photographic data (Ceplecha, 1961)
Fireball from the Příbram bolide, cutting across star trails, photographed by the Czech meteor network in 1959. The image shows the first part of the trajectory, from 97.8 to 68 km altitude. Chops in the fireball result from a rotating shutter above the lens, and allow the velocity to be calculated.
The largest piece (Luhy, 4.5 kg)
The largest programs

<table>
<thead>
<tr>
<th></th>
<th>European Fireball Network</th>
<th>Prairie (Meteorite) Network</th>
<th>Meteorite Observation &amp; Recovery Project</th>
</tr>
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<tbody>
<tr>
<td>Number of Stations</td>
<td>~ 50</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>Station Spacing, km</td>
<td>~ 90</td>
<td>250</td>
<td>193</td>
</tr>
<tr>
<td>Cameras per Station</td>
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<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Availability of obtained data</td>
<td>partly published</td>
<td>published</td>
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</tr>
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</table>
Immediate objectives

- more reliable and rapid meteorite recovery from the Earth’s surface
- understanding of meteorite origins, physical properties and chemical composition
- meteor orbital characteristics and their association with parent objects
- possible identification of when and where a meteor can enter the Earth’s atmosphere and potentially become a meteorite
- understanding of how to predict the consequences based on event observational data
## Registered meteorite falls

<table>
<thead>
<tr>
<th>Meteorite name</th>
<th>country</th>
<th>year</th>
<th>mass found</th>
<th>type</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Příbram</strong></td>
<td>Czechoslovakia</td>
<td>1959</td>
<td>5.8</td>
<td>H5</td>
</tr>
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<td>17.2</td>
<td>H5</td>
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<td>L5</td>
</tr>
<tr>
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<td>USA</td>
<td>1992</td>
<td>12.57</td>
<td>H6</td>
</tr>
<tr>
<td>Tagish Lake</td>
<td>Canada</td>
<td>2000</td>
<td>~ 10</td>
<td>CI</td>
</tr>
<tr>
<td>Morávka</td>
<td>Czech Republic</td>
<td>2000</td>
<td>1.4</td>
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<tr>
<td><strong>Neuschwanstein</strong></td>
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<td>2002</td>
<td>6.2</td>
<td>EL6</td>
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<tr>
<td>Park Forest</td>
<td>USA</td>
<td>2003</td>
<td>18</td>
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<tr>
<td>Villalbeto de la Peña</td>
<td>Spain</td>
<td>2004</td>
<td>5</td>
<td>L6</td>
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<td><strong>Bunburra Rockhole</strong></td>
<td>Australia</td>
<td>2007</td>
<td>0.34</td>
<td>eucrite</td>
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<tr>
<td>Buzzard Coulee</td>
<td>Canada</td>
<td>2008</td>
<td>~ 41</td>
<td>H4</td>
</tr>
<tr>
<td>Almahata Sitta</td>
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<td>2008</td>
<td>~ 4</td>
<td>ureilite</td>
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<td>L6</td>
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<tr>
<td>Košice</td>
<td>Slovakia</td>
<td>2010</td>
<td>3.92</td>
<td>H5</td>
</tr>
</tbody>
</table>
- Introduction
- A new method implemented for parameters’ identification
- Result testing
- General statistics and consequences
Groundbased observations

The information on meteor body entry into the atmosphere contains detailed dynamic and photometric observational data. The important input parameters are: the fireball brightness $I(t)$, its height $h(t)$ and its velocity $V(t)$.
Entry starts with a rarefied hypersonic flow (usually defined as those with a Knudsen number above 0.1), do not follow the Navier-Stokes equations. The introduction of non-equilibrium real gas effects becomes important.
Interpretation of Earth observations

**Photometric**

\[ I = -\tau \cdot \frac{dE}{dt} \]

Usually simplified case used is:

\[ \frac{dV}{dt} = 0 \]

\[ M = -\int_{t_1}^{t_0} \frac{I}{\tau V^2} dt \]

**Dynamical**

\[ M \frac{dV}{dt} = -\frac{1}{2} c_d\rho_a V^2 S, \]

\[ \frac{dh}{dt} = -V \sin \gamma, \]

\[ H \ast \frac{dM}{dt} = -\frac{1}{2} c_h\rho_a V^3 S \]
A dynamical methods: special case

\[
M \frac{dV}{dt} = -\frac{1}{2} c_d \rho_a V^2 S,
\]

\[
A = \frac{S \cdot \rho_b^{2/3}}{M^{2/3}}
\]

Halliday et al.

\[
M_d = \frac{(A \cdot c_d)^3}{\rho_b^2} \left( -\frac{\rho_a V^2}{2\dot{V}} \right)^3
\]

Wetherill, ReVelle

\[
M_d = 0.306 \left( -\frac{\rho_a V^2}{2\dot{V}} \right)^3
\]

\[
M_d = 0.101 \left( -\frac{\rho_a V^2}{2\dot{V}} \right)^3
\]

- The main drawback is approximation of constant meteoroid shape, which adopted in the majority of publications in Meteor Physics
- The values of deceleration are obtained with the numerical differentiation
More general approach

\[ m \frac{dv}{dy} = \frac{1}{2} c_d \frac{\rho_0 h_0 S_e}{M_e} \frac{\rho v s}{\sin \gamma}; \quad \frac{dm}{dy} = \frac{1}{2} c_h \frac{\rho_0 h_0 S_e}{M_e} \frac{V_e^2}{H} \frac{\rho v^2 s}{\sin \gamma} \]

- \( m = M/M_e; M_e \) – pre-atmospheric mass
- \( v = V/V_e; V_e \) – velocity at the entry into the atmosphere
- \( y = h/h_0; h_0 \) – height of homogeneous atmosphere
- \( s = S/S_e; S_e \) – middle section area at the entry into the atmosphere
- \( \rho = \rho_d/\rho_0; \rho_0 \) – gas density at sea level
Two additional equations

- variations in the meteoroid shape can be described as (Levin, 1956)
  \[ \frac{S}{S_e} = \left( \frac{M}{M_e} \right)^\mu \]

- assumption of the isothermal atmosphere
  \[ \rho = \exp(-y) \]
Analytical solutions of dynamical eqs.

Initial conditions

\[ y = \infty, \quad v = 1, \quad m = 1 \]

\[ m(v) = \exp\left( -\beta \frac{1-v^2}{1-\mu} \right) \]

\[ y(v) = \ln 2\alpha + \beta - \ln(\overline{Ei}(\beta) - \overline{Ei}(\beta v^2)) \]

where by definition:

\[ \overline{Ei}(x) = \int_{-\infty}^{x} \frac{e^z}{z} \, dz \]
The key dimensionless parameters used:

\[ \alpha = \frac{1}{2} c_d \frac{\rho_0 h_0 S_e}{M_e \sin \gamma}, \quad \beta = \left(1 - \mu\right) \frac{c_h V_e^2}{2 c_d H^*}, \quad \mu = \log_m s \]

\(\alpha\) characterizes the aerobraking efficiency, since it is proportional to the ratio of the mass of the atmospheric column along the trajectory, which has the cross section \(S_e\), to the body’s mass.

\(\beta\) is proportional to the ratio of the fraction of the kinetic energy of the unit body’s mass to the effective destruction enthalpy.

\(\mu\) characterizes the possible role of the meteoroid rotation in the course of the flight.
Next step: determination of $\alpha$ and $\beta$

On the right:
Data of observations of Innisfree fireball (Halliday et al., 1981)

\[ y(v) = \ln 2\alpha + \beta + -\ln(\overline{Ei}(\beta) - \overline{Ei}(\beta v^2)) \]

\[ \overline{Ei}(x) = \int_{-\infty}^{x} \frac{e^z}{z} dz \]

The problem is solved by the least squares method

<table>
<thead>
<tr>
<th>$t$, sec</th>
<th>$h$, km</th>
<th>$V$, km/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,0</td>
<td>58,8</td>
<td>14,54</td>
</tr>
<tr>
<td>0,2</td>
<td>56,1</td>
<td>14,49</td>
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<td>0,4</td>
<td>53,5</td>
<td>14,47</td>
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<td>0,6</td>
<td>50,8</td>
<td>14,44</td>
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<td>0,8</td>
<td>48,2</td>
<td>14,40</td>
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<tr>
<td>1,0</td>
<td>45,5</td>
<td>14,34</td>
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<td>1,2</td>
<td>42,8</td>
<td>14,23</td>
</tr>
<tr>
<td>1,4</td>
<td>40,2</td>
<td>14,05</td>
</tr>
<tr>
<td>1,6</td>
<td>37,5</td>
<td>13,79</td>
</tr>
<tr>
<td>1,8</td>
<td>35,0</td>
<td>13,42</td>
</tr>
<tr>
<td>2,0</td>
<td>32,5</td>
<td>12,96</td>
</tr>
<tr>
<td>2,2</td>
<td>30,2</td>
<td>12,35</td>
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<td>2,4</td>
<td>27,9</td>
<td>11,54</td>
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<td>2,6</td>
<td>25,9</td>
<td>10,43</td>
</tr>
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<td>2,8</td>
<td>24,2</td>
<td>8,89</td>
</tr>
<tr>
<td>3,0</td>
<td>22,6</td>
<td>7,24</td>
</tr>
<tr>
<td>3,2</td>
<td>21,5</td>
<td>5,54</td>
</tr>
<tr>
<td>3,3</td>
<td>21,0</td>
<td>4,70</td>
</tr>
</tbody>
</table>
The desired parameters are determined by the following formulas:

**The necessary condition for extremum:**

\[
\alpha = \sum_{i=1}^{n} e^{-\beta - y_i} \cdot \Delta_i \bigg/ 2 \sum_{i=1}^{n} e^{-2y_i};
\]

\[
\sum_{i=1}^{n} \left[ \Delta_i \left( \sum_{i=1}^{n} \exp(-2y_i) \right) - \left( \sum_{i=1}^{n} \Delta_i \exp(-y_i) \right) \exp(-y_i) \cdot (\Delta_i - (\Delta_i)_\beta) \right] = 0
\]

**The sufficient condition:**

\[
\sum_{i=1}^{n} e^{-2y_i} \sum_{i=1}^{n} (((\Delta_i)_\beta - \Delta_i)^2 + (\Delta_i - 2\alpha \exp(\beta - y_i))(\Delta_i)_\beta - 2(\Delta_i)_\beta + \Delta_i) \left( \sum_{i=1}^{n} \exp(-y_i)(\Delta_i - (\Delta_i)_\beta) \right)^2 > 1
\]
Final step: determination of $\mu$

- From the current dynamical model the intensity of fireball luminosity can be rewritten as:

$$ I = -\tau \cdot \frac{d(MV^2/2)}{dt} = -\tau \left( \frac{V^2}{2} \frac{dM}{dt} + MV \frac{dV}{dt} \right) = $$

$$ = -\tau \cdot \left( M_e V_e \frac{\beta v^3}{1-\mu} + M_e V_e v \right) \exp \left( -\beta \frac{1-v^2}{1-\mu} \right) \frac{dV}{dt} = $$

$$ = \tau \cdot \frac{M_e V_e^3 \sin \gamma}{2h_0} v^3 \cdot (\text{Ei}(\beta) - \text{Ei}(\beta v^2)) \cdot \left( \frac{\beta v^2}{1-\mu} + 1 \right) \cdot \exp \left( \beta \frac{\mu v^2 - 1}{1-\mu} \right) $$

- This can be compared with values $I(v)$ calculated from the observed magnitude
The dependence between light curve and shape change coefficient $\mu$
Introduction

A new method for parameters’ identification

Result testing

General statistics and consequences
Result of calculations for well registered meteorite falls

<table>
<thead>
<tr>
<th>fireball</th>
<th>$V_e$, km/s</th>
<th>$\sin \gamma$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$10^2 \cdot \sigma$, s$^2$/km$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Příbram</td>
<td>20,887</td>
<td>0,68</td>
<td>8,34</td>
<td>13,64</td>
<td>6,25</td>
</tr>
<tr>
<td>Lost City</td>
<td>14,1485</td>
<td>0,61</td>
<td>11,11</td>
<td>1,16</td>
<td>1,16</td>
</tr>
<tr>
<td>Innisfree</td>
<td>14,54</td>
<td>0,93</td>
<td>8,25</td>
<td>1,70</td>
<td>1,61</td>
</tr>
<tr>
<td>Neuschwanstein</td>
<td>20,95</td>
<td>0,76</td>
<td>3,92</td>
<td>2,57</td>
<td>1,17</td>
</tr>
</tbody>
</table>
Analytical solution with parameters found by the method (solid lines) is compared to observation results (dots)
Meteoroid’s Deceleration

\[ y(v) = \ln 2\alpha + \beta - \ln \left( \frac{Ei(\beta)}{Ei(\beta v^2)} \right) \]

\[ \frac{dy}{dt} = -\frac{V_e}{h_0} v \sin \gamma \]

\[ \frac{dV}{dt} = V_e \frac{dv}{dy} \frac{dy}{dt} = -\frac{V_e^2 v^2 \sin \gamma}{2h_0} \frac{Ei(\beta) - Ei(\beta v^2)}{\exp(\beta v^2)} \]

We can compare data in the planes \((t, V)\) and \((t, h)\)
Coupling of parameters used in meteoroid entry modelling

Lost City: velocity of the main body

Comparison with paper Ceplecha, ReVelle 2005
Lost City: height of the main body

Comparison with paper Ceplecha, ReVelle 2005
Lost City: deceleration of the main body

Comparison with paper Ceplecha, ReVelle 2005
Terminal mass prediction: comparison with found meteorites’ mass

<table>
<thead>
<tr>
<th>fireball</th>
<th>ρ, g/cm³</th>
<th>Vₜ, km/s</th>
<th>M(Vₜ), kg</th>
<th>Mₜ, kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lost City</td>
<td>3,73</td>
<td>3,4</td>
<td>43</td>
<td>17,2</td>
</tr>
<tr>
<td>Innisfree</td>
<td>3,5</td>
<td>4,7</td>
<td>27</td>
<td>4,58</td>
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<tr>
<td>Neuschwanstein</td>
<td>3,49</td>
<td>2,3</td>
<td>28</td>
<td>6,2</td>
</tr>
</tbody>
</table>
Our result corresponds to the estimates done based on the analysis of recorded acoustic, infrasound and seismic waves caused by the meteorite fall.
### Initial masses for earlier meteorite falls

<table>
<thead>
<tr>
<th></th>
<th>Příbram</th>
<th>Lost City</th>
<th>Innisfree</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, kg</td>
<td>317</td>
<td>169</td>
<td>179</td>
</tr>
<tr>
<td>M¹, kg</td>
<td>1300</td>
<td>52</td>
<td>18</td>
</tr>
<tr>
<td>M₂, kg</td>
<td>21500²</td>
<td>490³</td>
<td>318⁴</td>
</tr>
<tr>
<td>M³, kg</td>
<td>320⁵</td>
<td>65⁵</td>
<td>?</td>
</tr>
<tr>
<td>M⁴, kg</td>
<td>–</td>
<td>165</td>
<td>42</td>
</tr>
<tr>
<td>M⁵, kg</td>
<td>250⁷</td>
<td>210⁷</td>
<td>–</td>
</tr>
<tr>
<td>M⁶, kg</td>
<td>1700</td>
<td>38</td>
<td>25</td>
</tr>
</tbody>
</table>

Characteristic heights for the fireball trajectories

\[ L = \left( \frac{M_e}{30 \rho_m} \right)^{\frac{1}{3}}, \quad R = \left( \frac{30 L^3}{4 \pi / 3} \right)^{\frac{1}{3}} \]

- \( h_t \) – the height at which the size \( L \) is equal to the mean free path of air molecules
  \[ h_t = h_0 \ln \left( \frac{L}{l_0} \right), \quad l_0 = 0.19 \cdot 10^{-4} \text{ см} \]

- \( h_{sw} \) – the height below which the flow around the equivalent sphere of radius \( R \) takes place in the thin viscous shock layer regime (i.e. a thin shock is formed)
  \[ h_{sw} = 53 + 17.05 \lg R \]

- \( h_{bl} \) – the height corresponding to the formation of a thin boundary layer on the equivalent sphere
  \[ h_{bl} = 40.7 + 15 \lg R \]
## Calculated characteristic heights

<table>
<thead>
<tr>
<th>№</th>
<th>$h_b$, km</th>
<th>$h_{ml}$, km</th>
<th>$h_t$, km</th>
<th>$M_e$, kg</th>
<th>L, cm</th>
<th>$h_l$, km</th>
<th>R, cm</th>
<th>$h_{sw}$, km</th>
<th>$h_{bl}$, km</th>
</tr>
</thead>
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<tr>
<td>18</td>
<td>75,5</td>
<td><strong>44,9</strong></td>
<td>27,5</td>
<td>11,5</td>
<td>4,8</td>
<td>89,0</td>
<td>9,2</td>
<td>69,4</td>
<td><strong>55,2</strong></td>
</tr>
<tr>
<td>169</td>
<td>78,9</td>
<td><strong>44,2</strong></td>
<td>34,0</td>
<td>6,1</td>
<td>3,9</td>
<td>87,5</td>
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<td>67,9</td>
<td><strong>53,8</strong></td>
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<tr>
<td>195</td>
<td>77,4</td>
<td><strong>40,6</strong></td>
<td>30,4</td>
<td>3,3</td>
<td>3,2</td>
<td>86,1</td>
<td>6,1</td>
<td>66,4</td>
<td><strong>52,5</strong></td>
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<tr>
<td>204</td>
<td>61,9</td>
<td><strong>40,6</strong></td>
<td>29,5</td>
<td>187,9</td>
<td>12,1</td>
<td>95,7</td>
<td>23,4</td>
<td>76,3</td>
<td><strong>61,2</strong></td>
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<tr>
<td>205</td>
<td>72,5</td>
<td><strong>38,0</strong></td>
<td>28,9</td>
<td>0,6</td>
<td>1,8</td>
<td>81,9</td>
<td>3,4</td>
<td>62,1</td>
<td><strong>48,7</strong></td>
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<tr>
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<td>26,1</td>
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<td>6,8</td>
<td>91,6</td>
<td>13,2</td>
<td>72,1</td>
<td><strong>57,5</strong></td>
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<tr>
<td>223</td>
<td>78,5</td>
<td><strong>49,0</strong></td>
<td>27,1</td>
<td>141,7</td>
<td>11,1</td>
<td>95,0</td>
<td>21,3</td>
<td>75,6</td>
<td><strong>60,6</strong></td>
</tr>
<tr>
<td>276</td>
<td>81,8</td>
<td><strong>32,4</strong></td>
<td>24,4</td>
<td>8,2</td>
<td>4,3</td>
<td>88,2</td>
<td>8,3</td>
<td>68,6</td>
<td><strong>54,4</strong></td>
</tr>
<tr>
<td>285</td>
<td>58,8</td>
<td><strong>35,0</strong></td>
<td>19,8</td>
<td>104,1</td>
<td>10,0</td>
<td>94,3</td>
<td>19,2</td>
<td>74,9</td>
<td><strong>60,0</strong></td>
</tr>
</tbody>
</table>
- Introduction
- A new method implemented for parameters’ identification
- Result testing
- General statistics and consequences
Distribution of parameters $\alpha$ and $\beta$ for MORP fireballs
Looking for Meteorite ‘region’

\[ M_t \geq M_{\text{min}}, \quad v_t \ll 1 \]

\[ \beta + 3\ln \alpha = -\ln m_{\text{min}} \]

\[ \sin \gamma = 0.7 \text{ and } 1; \quad M_{\text{min}} = 8 \text{ kg} \]
Meteorite falls prediction (down to 50 g)

\[
\sin \gamma = 0.7 \text{ and } 1; \ M_{\text{min}} = 8 \text{ kg and } 0.05 \text{ kg}
\]
### Some historical events

<table>
<thead>
<tr>
<th>№</th>
<th>Event</th>
<th>Original mass, $t$</th>
<th>Collected meteorites, $kg$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Canyon Diablo meteorite</td>
<td>$&gt; 10^6$</td>
<td>$&gt; 30 \cdot 10^3$</td>
<td>0.1</td>
<td>$\sim 0.1$</td>
</tr>
<tr>
<td>2</td>
<td>Tunguska</td>
<td>$0.2 \cdot 10^6$</td>
<td>-</td>
<td>0.3</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>Sikhote Alin</td>
<td>200</td>
<td>$&gt; 28 \cdot 10^3$</td>
<td>1.2</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>Neuschwanstein</td>
<td>0.5</td>
<td>6.2</td>
<td>3.9</td>
<td>2.5</td>
</tr>
<tr>
<td>5</td>
<td>Benešov</td>
<td>0.2</td>
<td>-</td>
<td>7.3</td>
<td>1.8</td>
</tr>
<tr>
<td>6</td>
<td>Innisfree</td>
<td>0.18</td>
<td>4.58</td>
<td>8.3</td>
<td>1.7</td>
</tr>
<tr>
<td>7</td>
<td>Lost City</td>
<td>0.17</td>
<td>17.2</td>
<td>11.1</td>
<td>1.2</td>
</tr>
</tbody>
</table>
Same events on the plane ($\ln \alpha$, $\ln \beta$)

- Crater Barringer
- Tunguska
- Sikhote Alin
- Neuschwanstein
- Benešov
- Innisfree
- Lost City

- $\sim 1,2$ km
Same events on the plane \((\ln \alpha, \ln \beta)\)

<table>
<thead>
<tr>
<th>Event</th>
<th>Corresponding Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crater Barringer</td>
<td>(1)</td>
</tr>
<tr>
<td>Tunguska</td>
<td>(2)</td>
</tr>
<tr>
<td>Sikhote Alin</td>
<td>(3)</td>
</tr>
<tr>
<td>Neuschwanstein</td>
<td>(4)</td>
</tr>
<tr>
<td>Benešov</td>
<td>(5)</td>
</tr>
<tr>
<td>Innisfree</td>
<td>(6)</td>
</tr>
<tr>
<td>Lost City</td>
<td>(7)</td>
</tr>
</tbody>
</table>

- \(0 < \alpha < 1, \beta > 1\)
- \(\alpha > 1, \beta > 1\)
- \(0 < \alpha < 1, 0 < \beta < 1\)
- \(\alpha > 1, 0 < \beta < 1\)

\(-1.2 \text{ km}\)
Meteorites /craters prediction

<table>
<thead>
<tr>
<th></th>
<th>Crater Barringer</th>
<th>Tunguska</th>
<th>Sikhote Alin</th>
<th>Neuschwanstein</th>
<th>Benešov</th>
<th>Innisfree</th>
<th>Lost City</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
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<td>7</td>
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<td></td>
</tr>
</tbody>
</table>

\[ \ln \beta \]

\[ \ln \alpha \]

~1.2 km
Conclusions

Consideration of non-dimensional parameters $\alpha$, $\beta$ and $\mu$ allow us to predict consequences of the meteor impact. These parameters effectively characterize the ability of entering body to survive an atmospheric entry and reach the ground. The set of these parameters can be also used to solve inverse problems, when we have to evaluate properties of the entering body based on observations.

The results are applicable to study the properties of near-Earth space and can be used to predict and quantify fallen meteorites, and thus to speed up recovery of their fragments.
Some relevant papers


Thank you very much!
Why do we pass from $y$ to $\exp(-y)$?

$$e^{-y} = \frac{\Delta \cdot e^{-\beta}}{2\alpha}, \quad \Delta = \text{Ei}(\beta) - \text{Ei}(\beta v^2), \quad \text{Ei}(x) = \int_{-\infty}^{x} \frac{e^z dz}{z}$$
Newton's Second Law of Motion

\[
M \frac{dV}{dt} = -\frac{1}{2} c_d \rho_a V^2 S + Mg \sin \gamma
\]

\[
\frac{dM \vec{V}}{dt} = \lim_{\Delta t \to o} \frac{\Delta M \vec{V}}{\Delta t}
\]

\[
\Delta M \vec{V} = \left[ (M - \Delta M)(\vec{V} + \Delta \vec{V}) + \Delta M (\vec{V} + \vec{U}) \right] - M \vec{V}
\]

\[
\frac{dM \vec{V}}{dt} = \lim_{\Delta t \to o} \frac{\Delta M \vec{V}}{\Delta t} = \lim_{\Delta t \to o} \frac{M \Delta \vec{V} + \Delta M \vec{U}}{\Delta t} = M \frac{d\vec{V}}{dt}
\]
Mass computation

\[ M_e = \left( \frac{1}{2} c_d \frac{\rho_0 h_0}{\alpha \sin \gamma} \frac{A_e}{\rho_m^{2/3}} \right)^3 \]

Initial Mass depends on ballistic coefficient

\[ M(v) = \left( \frac{1}{2} c_d \frac{\rho_0 h_0}{\alpha \sin \gamma} \frac{A_e}{\rho_m^{2/3}} \right)^3 \cdot \exp \left( -\frac{\beta}{1-\mu} (1-v^2) \right) \]
Forms of some recovered meteorites

- Příbram
- Innisfree
Определение внеатмосферной массы

\[ \alpha = \frac{1}{2} c_d \frac{\rho_0 h_0 S_e}{M_e \sin \gamma} \]

\[ M_e = \left( \frac{1}{2} c_d \frac{\rho_0 h_0 A_e}{\rho_m^{2/3}} \frac{1}{\alpha \sin \gamma} \right)^3 \]

gде \( A_e = \frac{S_e}{W_e^{2/3}} \) коэффициент начальной формы тела

Используемые параметры

- плотность атмосферы на уровне моря \( \rho_0 \),
- высота однородной атмосферы \( h_0 \),
- угол траектории с горизонтом \( \gamma \)
- коэффициент формы тела при входе в атмосферу \( A_e \)
- коэффициент сопротивления \( c_d \),
- плотность тела \( \rho_m \),
Определение динамических параметров

1. Преобразование точного аналитического решения к виду:

\[ 2\alpha \exp(-y) - \Delta \exp(-\beta) = 0, \quad \Delta = \operatorname{Ei}(\beta) - \operatorname{Ei}(\beta v^2) \]

2. Определение величины расхождения данных наблюдений и теоретической зависимости

\[ F(y_i,v_i,\alpha,\beta) = 2\alpha \exp(-y_i) - \Delta_i \exp(-\beta) \]

3. Применение метода наименьших квадратов для определения параметров \( \alpha \) и \( \beta \)

\[ Q_4(\alpha,\beta) = \sum_{i=1}^{n} (F(y_i,v_i,\alpha,\beta))^2 \rightarrow \text{min} \]